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Automated soil inference under fuzzy logic

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Abstract

Soil information is essential to any terrestrial ecological modelling and management activity. Polygon soil maps produced from soil surveys are currently the major source of information on the spatial distribution of soil properties for a variety of land analysis and management activity. However, there are some major problems regarding the use of current soil maps in geographic analysis and especially in geographic information systems (GIS). These problems include limited coverage at a fixed scale, locational errors, attribute errors, and insufficient information in the mapping units. Much of these problems are due to the crisp logic and cartographic techniques with which soil maps are produced. Under crisp logic standardly used in soil classification and mapping, an area belongs to one and only one soil mapping unit, and is separated from other mapping units by sharp boundary lines. However, soil in a landscape is a continuum and the discretization of such a continuum into distinct spatial and categorical groups results in a significant loss of information.

This paper presents a methodology to infer and represent information on the spatial distribution of soil. The methodology combines fuzzy logic with GIS and expert system development techniques to infer soil series from environmental conditions. The methodology for every point in an area produces a soil similarity vector (SSV) showing the similarity of the soil at the point to a prescribed set of soil series. The SSV produced from this methodology can be used to infer local soil properties at values intermediate to the typical or central values assigned to each possible series. Preliminary results from the methodology using a limited set of environmental variables for an experimental watershed in Montana are presented.

Keywords: Fuzzy logic; Soil ecosystems; Geographical information system; Expert systems

1. Introduction

Knowledge of the distribution of soil properties over the landscape is required for a variety of hydrological, ecological and land management applications. In detailed hydroecological and other environmental modelling applications, continuous soil prop-

erties over an area are very much desired to approximate the resolution of other environmental parameters gathered from remote sensing and digital terrain analysis (Band et al., 1991, 1993). Unfortunately, information on the spatial patterns of soil properties is very difficult to directly obtain over large areas as soils show inherently high and gradual spatial variation, and are often obscured by a vegetation canopy.

This paper presents a methodology to infer and represent information on the spatial distribution of soil. The methodology combines fuzzy logic with

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GIS and expert system development techniques to infer soil series from environmental conditions. The methodology for every point in an area produces a soil similarity vector (SSV) showing the similarity of the soil at the point to a prescribed set of soil series. In this way, local soil properties may be inferred at values intermediate to the typical or central values assigned to each possible series.

Currently, soil maps produced from soil surveys are digitized to provide the soil information required for a variety of land analysis and management activity. However, soil maps often contain a great deal of uncertainty as much of the quantitative and qualitative knowledge of the soil scientist regarding the occurrence of given soil categories (e.g. series) or properties is not maintained in the map. There are some major problems regarding the use of current soil maps in geographic analysis (Burrough, 1986) including limited coverage at a fixed scale, locational errors, attribute errors, and insufficient information in the mapping units due to the crisp logic and cartographic techniques with which soil maps are produced. These problems are particularly severe for areas which are non-agricultural lands because soil surveys are less intensive or do not exist at all.

The scale of the soil map determines the spatial resolution of soil variation to be mapped. Soil mapping units are rarely single category units, even on large-scale soil maps. This means that an area which is mapped as the same soil mapping unit could include many different soil taxonomic types in a mapped complex (mixed category unit). This mixed category unit problem is due to the cartographic technique used in the mapping processes. At a certain scale, only soil objects larger than certain size (scale dependent) can be represented on the soil maps. Therefore, the knowledge of soil scientists about soil variation cannot be fully represented by soil maps.

There are two interrelated types of errors in soil maps: locational errors and attribute errors. Locational errors are introduced into soil maps by improper positioning of boundaries between soil bodies. The introduction of locational errors is not due purely to the mistakes made by soil mapping experts but also due to the nature of soil boundaries. Soil varies gradually and the boundaries between different types of soils are often diffused rather than sharp

(Mark and Csillag, 1990). However, soils have to be delineated into homogeneous polygons on soil maps. Therefore, it is difficult for any soil mapping expert to draw a boundary between two soils without introducing locational errors. The attribute error arises when an area mapped as one soil map unit may not be uniform in terms of properties described in the map legend. This error is considered as attribute error.

Studies have shown that even soil maps produced from systematic surveys may contain large amounts of 'impurities' (errors) within the units delineated (Wilding et al., 1965; Beckett, 1971; Beckett and Burrough, 1971). These 'impurities' may propagate through geographical analysis in a GIS and make the results from these analysis, particularly GIS-based simulation modelling, unpredictable. While the soil scientists conducting the survey and mapping process may be aware of the degree of uncertainty in the mapping, and may be able to infer more detail on expected soil properties at a given location on the map from their knowledge of soil-landscape relations, this information is generally not translated onto the soil map. Therefore, there has developed an interest in alternate methods of both gathering information on the spatial distribution of soils, and on the encoding of that information in a form suitable to GIS that will incorporate uncertainty in both the location of given soil types and in their properties.

Goodchild et al. (1992) developed an error model for categorical data. In their model, for each pixel (i, j), an n -elements probability vector (P) was assigned. Element k , (p^k), in the vector represents the probability of the pixel falling into soil category k . Together with a spatial dependence parameter (ρ) they used the model to estimate the area of a particular soil type in a given soil polygon using a priori information on p^k and ρ . This is a very valuable approach for estimating the area of particular category in the map. However, in our context we assume no prior information on soil frequency or spatial dependence, and no existing soil map. We are concerned with the similarities of the soil at a point to a set of known soil taxonomic units.

Burgess and Webster (1980a,b), Webster and Burgess (1980), Webster and Oliver (1989), Webster (1991), Loague and Gander (1990) and Loague (1992) have explored the usefulness of geostatistical

methods in measuring the spatial variability of some soil attributes. These geostatistical methods are very useful for simple landscapes that can satisfy stationarity assumptions of geostatistics. However, these quantitative interpolation techniques may have limited usage for complex terrain where pedogenesis arises in a complex manner and the stationarity assumptions of geostatistics may not be met. These techniques also require a large set of field sampled data, which is often not available for many applications. Moore et al. (1993) used multiple linear regression analysis to relate soil properties to topographic attributes and used the relationship to predict soil properties. The technique assumes that the relationship between soil and topographic attributes is linear and it requires a great deal of field data to extract the relationship. Because of its assumption and the data requirement, the technique has very limited practical use. Skidmore et al. (1991) employed an expert system approach to infer soil-landscape units from four data layers (forest overstorey, gradient, topographical position and soil wetness index). The expert system is based on a Bayesian inference technique. However, the inference was once again done under crisp logic and used to produce crisp soil-landscape maps similar to conventional soil maps.

Burrough et al. (1992) used fuzzy classification to determine land suitability from multivariate point observations of soil attributes, topographically controlled site drainage conditions, and minimum contiguous areas. In their study, the emphasis was placed on the attribute rather than the spatial distribution of land suitability. Odeh et al. (1992) used Fuzzy-*c*-means classifier to identify fuzzy soil classes among the soil profiles sampled from two transects. They found that these fuzzy soil classes were very strongly associated with various landforms. The study, however, did not show how these relationships between landforms and fuzzy soil classes can be used to produce continuous soil property maps.

We consider that the soil at any point or location is similar, in varying degrees, to a prescribed set of soil taxonomic units (such as soil series) or central concepts. Soil at any point or location, (i, j) , can be expressed by an n -element vector, $S_{ij} = (s_{ij}^1, s_{ij}^2 \dots s_{ij}^k \dots s_{ij}^n)$, where s_{ij}^k is a similarity measure (or membership) of the soil at point (i, j) to the pre-

scribed soil taxonomic unit k and n is the number of soil taxonomic units in the area. We call vector S the soil similarity vector (SSV). Thus, SSV at point (i, j) will be represented as SSV_{ij} . This representation of soil information is different from the conventional crisp representation. The similarity of the soil at a location to a soil taxonomic unit is expressed in terms of a membership value between 0.0 and 1.0 (that is, $0.0 \leq s_{ij}^k \leq 1.0$), and not a yes or no. It should be clarified that s_{ij}^k is not probability but a fuzzy membership which used to express the similarity of the soil at point (i, j) to the prescribed taxonomic unit k . It is this membership value that will provide users with the information about the similarity (or confidence) of the soil at a point to soil taxonomic unit k .

The aim of this paper is to present a soil inference model (SOLIM, Soil Land Inference Model) for deriving these SSVs. A subsequent paper will use these SSVs to derive soil properties. In our inference model, we combine empirical knowledge on pedogenesis with information of the soil environment derived through a set of GIS techniques to infer SSVs.

In this illustration we use soil series as our taxonomic unit describing the fuzzy sets to which a soil at a given location will have fuzzy memberships. The reason for using soil series as our basic taxonomic units is that soil series is the taxonomic unit which has been extensively used in soil surveys and local soil experts feel more comfortable with soil series than any other taxonomic units. In this illustration, we employed six environmental factors (elevation, aspect, gradient, canopy coverage, parent material, and surface profile curvature) and a knowledge set on the relationships between these factors and four soil series (Ambrant, Elkner, Ovando, and Rochester soil series) in western Montana (details about the study area and the four soil series are described in Section 4). We then combined these six environmental factors with the knowledge set to infer the SSVs over an area.

The raster data model is chosen to represent data layers and results in our method because the raster model is more suitable for representing continuous spatial variation of soil. The resolution of a raster model depends on the input environmental data (such as Digital Elevation Model (DEM), and remotely sensed data). In other words, the method is capable

of representing soil information at pixel resolution and is not limited by the minimum mapping area of a polygon based map. The method does not rely on field sampled soil data but such data when it does exist can be used to refine and enrich the knowledge base of the system and therefore improve the performance of the system.

In the next section, we discuss the theoretical basis of the methodology. This is then followed by the description of the methodology. In Section 4, we give a brief description of the study area and discuss the environmental variables employed. In Section 5, we present and discuss the results from the methodology. Summary and future direction of this methodology are presented in Section 6.

2. Theoretical basis for automated soil inference using fuzzy logic

2.1. Theoretical basis

The theoretical basis for soil inference is based on the classic concept of Jenny (1941, 1980) that a soil is a product of interaction among climatic factors, landform, parent material, organism, and hydrological factors over time. Therefore, we may infer the soil type at a given location if we have local environment conditions. This can be expressed in qualitative terms by

$$S = f(Cl, Og, Pm, Tp, t) \quad (1)$$

where *Cl* represents climate conditions, *Og* is for organism, *Pm* is parent material, *Tp* stands for topography, and *t* is time. Eq. 1 illustrates the general relationship between the soil and its environmental factors. However, the details of the relationship are different at different places. It is very difficult at this stage to derive a mathematical formula for the relationship because of the complexity and limited understanding of both soil forming processes and the paleo-environment. Over decades of study of soil-environment relationships, a great deal of empirical knowledge has been accumulated (for example, Armson, 1977; Jenny, 1980; Gerrard, 1981; Birke-land, 1984; Brady, 1984; Boul et al., 1989; Rendig and Taylor, 1989). Particularly, local soil scientists who study and map soils in their respective regions

have accumulated a detailed knowledge on soil-environment relationships. It is our belief that this empirical knowledge can be used to approximate relationship in Eq. 1 for soil series inference. We approach this approximation process with the use of a simple expert system and fuzzy logic.

2.2. Basics of expert system approach

Expert systems are software systems capable of representing and reasoning about some knowledge-rich domain with a view to solving problems and giving advice (Hall and Kandel, 1992). They derive their power from a great deal of domain-specific knowledge, rather than from a single powerful technique, which means the emphasis is firmly on knowledge itself rather than on formal reasoning methods. Thus based on a great deal of knowledge, expert systems are able to solve the domain specific problems (usually very difficult tasks).

Most expert systems are organized on three levels: data, knowledge base, and inference engine. The term 'data' in expert systems refers to information such as the environmental conditions of an area in our study, elevation, gradient, aspect, etc. The knowledge base contains the declarative knowledge about a particular problem being solved, particularly relationships among events or phenomena. For example, the knowledge base in our study contains the relationships between the environmental factors of a geographic region and soil series in that area. Expert systems separate this domain-specific knowledge from the procedural language (inference engine) by storing such knowledge in a knowledge base. This makes it much easier to encode and to maintain knowledge without affecting the execution mechanism of the program. The inference engine controls when and how specific problem-solving knowledge is used.

As expert systems are highly reliant on domain-specific knowledge, it is clear that the accuracy and sufficiency of domain-specific knowledge will determine the success of an expert system. The process of obtaining domain specific knowledge is called knowledge acquisition and is considered as the bottleneck of the development and application of effective expert systems (Gaines and Shaw, 1991). The

development of knowledge acquisition techniques has attracted a great deal of attention from the artificial intelligence (AI) community in the past few years (Greenwell, 1988; Brule and Blount, 1989; McGraw and Harbison-Briggs, 1989; McGraw and Westphal, 1990; Wielinga et al., 1990; Motoda et al., 1991; Scott et al., 1991). Many knowledge acquisition techniques have been developed. Some of these techniques were employed in this study to acquire empirical knowledge on soil–environment relationships from local soil scientist (see Section 3).

2.3. Basics of fuzzy set theory

Fuzzy logic is an infinite-valued logic which is different from the classic two-valued (yes or no) logic (crisp logic). Therefore, the membership in a fuzzy set is not characterized by ‘yes(1)’ or ‘no(0)’, but is more adequately considered in terms of degrees. As discussed in the introduction section of this paper, soil is a spatial continuum. Soil at a point resembles a prescribed soil series to certain extent but may not be the same as the prescribed soil series. Therefore, this characteristic of soil information leads itself to fuzzy logic representation.

A fuzzy set is characterized by a set of memberships, each of them is defined as a real number in the interval [0,1]. A formal definition of fuzzy set is given as follows.

Definition 1: Fuzzy Set (Zimmermann, 1985). If X is a collection of objects denoted generically by x , then a fuzzy set \tilde{A} in X is a set of ordered pairs:

$$A = \{x, \mu_{\tilde{A}}(x)\} \quad x \in X \quad (2)$$

where x is an object which belongs to the sets of objects X , $\mu_{\tilde{A}}(x)$ is the degree of membership, and $\mu_{\tilde{A}}()$ is the membership function of x in \tilde{A} which maps X to the membership space M . The membership function is very much dependent on the domain under study.

Because membership functions are the crucial components of a fuzzy set, the fuzzy set operations are defined via their membership functions. There are many ways to define the fuzzy set operations (Klir and Folger, 1988) and it is not the scope of this paper to discuss them all. We here discuss the basic fuzzy set operations.

Definition 2: Intersection. The membership function $\mu_{\tilde{N}}(x)$ of the intersection (logic ‘and’) set of fuzzy sets \tilde{A} and \tilde{O} is defined by

$$\mu_{\tilde{N}}(x) = \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{O}}(x)\}, \quad x \in X \quad (3)$$

Definition 3: Union. The membership function $\mu_{\tilde{N}}(x)$ of the union (logic ‘or’) set of fuzzy sets \tilde{A} and \tilde{O} is defined by

$$\mu_{\tilde{N}}(x) = \max\{\mu_{\tilde{A}}(x), \mu_{\tilde{O}}(x)\}, \quad x \in X \quad (4)$$

Definition 4: Complement. The membership function $\mu_{\tilde{N}}(x)$ of the complement (logic ‘not’) set of fuzzy set \tilde{A} is defined by

$$\mu_{\tilde{N}}(x) = \{1 - \mu_{\tilde{A}}(x)\}, \quad x \in X \quad (5)$$

Eqs. 3 and 4 are referred as the fuzzy minimum and maximum operators, respectively. There are many extensions to the above min–max definition to the fuzzy set operations (Zimmermann, 1985). However, discussion of these extensions is not appropriate here. Application of fuzzy mathematical methods in soil science and land evaluation has been very well discussed by many authors (Robinson, 1988; Burrough, 1989; Burrough et al., 1992; McBratney and De Grujter, 1992).

2.4. Assumptions of this study

As discussed in the introduction section, at any location, the soil will resemble, to a quantifiable extent, one or more of soil series and the soil at that point can be represented by an n -element SSV_{*ij*}. To derive SSV over an interested area, we first have to derive individual element of SSV over the area, that is, s^k for $k = 1 \dots n$, where s^k represent the fuzzy membership map of the area for soil series k . s^k is a $r \times c$ matrix, where r is number of rows (lines) and c is number of columns for the area. In other words, we first infer the similarity of the soil at every point (pixel) to a given soil series k in the area.

To derive s^k , we assume that every soil series occurs under one or more typical environmental configurations or ‘niches’ and has a typical set of

soil properties. The occurrence of soil series under these typical environment configurations is called the instance of the soil series in geographic space. For example, a given soil series (say, Soil Series A) may typically occur on south facing slopes at high elevations or on north facing slopes at low elevations due to its moisture requirement. Therefore, Soil Series A has two instances: one on south facing slopes at high elevations and the other on north facing slopes at low elevations. The environmental configuration of each of these instances of a given soil series, k , can be characterized by a vector of environmental parameters in an m -dimensional parameter space and can be represented by an m -element parameter vector, as $E_i^k = (e_{i1}^k \dots e_{il}^k \dots e_{im}^k)$, where e_{il}^k is the attribute value of l th environmental variable for the i th instance of soil series k . Inference of s^k then becomes two sub-problems: the problem of defining the instances of the soil series in the parameter space (typical or idealized environmental conditions), E^k , and the problem of determining the similarity of a soil to the soil series at a point away from the typical occurrences of the given soil series. In other words, the second problem addressing how the soil series varies in response to changes from its typical instances to other points in the parameter space (we refer this as the behaviour of a soil series). If we know the typical instances of a soil series and know the behaviour of the given soil series in the parameter space, we then are able to infer s_{ij}^k at any point (i, j) in the area of interest.

The typical instances, E^k , of a soil series in the parameter space can be determined from the soil series description and/or expert knowledge about the environmental conditions of soil series from soil experts. This knowledge is referred as 'Type 1' knowledge in this paper. Determining the behaviour of a given soil series in the parameter space with respect to the environmental condition changes is more difficult than determining the typical instances of the soil series. However, soil scientists working in a specific area may explicitly or implicitly understand how soils vary over the changes in environmental conditions in that area. It may be possible to use this part of local soil scientists' knowledge to approximate the behaviour of a given soil series over the landscape. Soil experts' knowledge on the behaviour of a soil series in response to the changes of

environmental conditions is referred here as 'Type 2' knowledge. As discussed above, both of these two types of knowledge can be obtained by some knowledge acquisition techniques (see Section 3). As outlined in Eq. 1, soil is a product of interaction of soil forming factors. In our case, there were six environmental variables involved (Section 4). It was not feasible to ask the soil experts to express Type 2 knowledge using the six variables simultaneously. Instead, as a first approximation we assume that each environmental factor has an influence on the formation of a soil that can be separately represented. Note that this may lead to some problems regarding the interaction of environmental conditions depending on the form of inference used and the number and nature of the instances of a given soil series. These potential problems will be further discussed below.

Under these assumptions, the soil experts formed the definitions of the soil series central concepts (instances) in terms of environmental conditions, and the influence of each of the environmental factors on the given soil series for each instance. We then integrated these influences to approximate the behaviour of the given soil series in response to the changes in environmental conditions. The influence was defined as the degree to which an environmental condition favours the development of a given soil series and was expressed as an optimality curve (Fig. 1). It was also assumed that the development of a soil series at a given location was controlled by the least optimal environmental condition for the soil series instance at that location. Therefore, the integration of these influences from the environmental

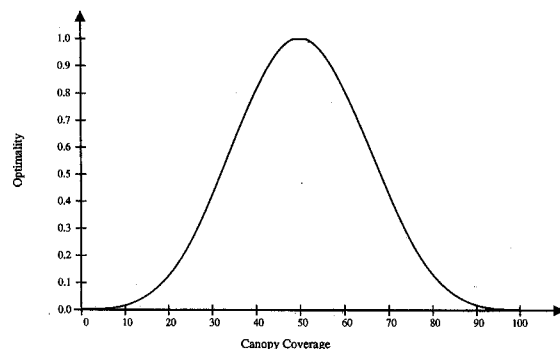


Fig. 1. Optimality of Ambrant soil series over canopy coverage for 'medium canopy coverage'.

variables can be modelled by the fuzzy intersection (fuzzy minimum operator).

In summary, an expert system approach and fuzzy set theory were used in this study to infer the soil similarity vector (SSV) over an area from soil environmental conditions. The employment of an expert system approach was to capture empirical knowledge on soil–environment relationship. Fuzzy set theory was employed in the inference process to model the interactions among the soil environment factors and the spatial continuity of soil series. Fuzzy set theory was also used to express the results from the inference processes.

3. Methodology

As discussed in Section 2, inference of s^k consists of two sub-problems: the problem of locating the typical occurrences of a particular soil series in the parameter space and the problem of determining the behaviours of the soil series in the parameter space. In this section, we discuss the methods for acquiring knowledge about the typical occurrences and behaviours of soil series in response to changes of environmental conditions. The details of actual inference are also outlined in this section.

3.1. Knowledge acquisition

As noted in Section 2, the knowledge about the relationships between soil series and its environmental conditions can be divided into two types. The first type (*Type 1* knowledge) regards where a soil series typically occurs in terms of environmental conditions. In other words, the first type of knowledge defines the instances for a given soil series. The second type (*Type 2* knowledge) regards the behaviours of a soil series to variation in the environmental conditions from its optimal conditions. Both of these two types of knowledge had to be extracted from local soil experts. In AI terminology, the people who perform the knowledge acquisition are called knowledge engineers and the experts whose knowledge is to be acquired are called domain experts.

Hoffman (1990) has discussed different methods of knowledge acquisition for expert systems. He

concludes that there are three broad categories of knowledge acquisition methods: task analysis methods, special task methods, and interview methods. With task analysis methods (also called methods of familiar task analyses), knowledge engineers study the tasks that the expert(s) usually perform. The specific task activities are decomposed and charted out step by step and are analyzed at whatever level of detail is sufficient for the purposes of the analysis. The knowledge engineers can also use special tasks as stimuli for a knowledge acquisition session to see experts' problem-solving behaviours. In any given domain, a special task will differ in some ways from the familiar tasks in that domain. Laboratory research on expertise indicates that deliberate departure from the familiar task can reveal the expert's knowledge and reasoning (Hoffman, 1990). Interview methods, as the name implies, take the form of dialogue between knowledge engineers and domain experts (Greenwell, 1988; McGraw and Harbison-Briggs, 1989; Scott et al., 1991). There are two types of interview methods: unstructured and structured. Unstructured interviews take the form of free-flowing dialogue in which open-ended questions are asked about the expert's knowledge and reasoning strategies. Structured interviews involve careful pre-planning of the questions and their order, and specification of things the knowledge engineers should do. Structured interviews reveals much of the experts' knowledge but can be very time-consuming. Many expert system developers used structured interview methods to elicit expert knowledge (Hoffman, 1990). However, these three types of techniques can be combined for knowledge acquisition.

In this study, we employed the structured interview method. We divided the knowledge acquisition process into four structured interviews: the soil–environment key development interview, the soil–environment description interview, the optimality curve definition interview, and knowledge verification interview (Zhu and Band, 1992).

3.1.1. Key development interview

The key development interview was to designate a dichotomous key to differentiate the soil series using the environmental variables. This was basically for the experts to clarify the difference between the

Table 1
Description of typical instances of the four soil series

Soil series	Instance	Parent material	Elevation (ft)	Aspect	Gradient (%)	Canopy coverage	Curvature
Ambrant	1	Granite	4000-5000	North	15-13	Medium	Convex to straight
Ambrant	2	Granite	4000-6000	South	15-60	Medium	Convex to straight
Elkner	1	Granite	> 4500	North	8-60	Medium	Convex to straight
Elkner	2	Granite	> 6000	South	8-60	Medium	Convex to straight
Ovando	1	Granite	> 4500	North	30-70	Medium to low	Convex to straight
Ovando	2	Granite	> 6000	South	30-70	Medium to low	Convex to straight
Rochester	1	Granite	4000-4500	North	30-70	Sparse	Convex to straight
Rochester	2	Granite	4000-6000	South	30-70	Sparse	Convex to straight

soil series in terms of the environmental variables and get them ready for the next interview, the description interview. During the key development interview, environment data were converted from met-

ric units to standard units with which our soil experts were more comfortable. A simple key to the four soil series developed during this key development interview is shown as follow.

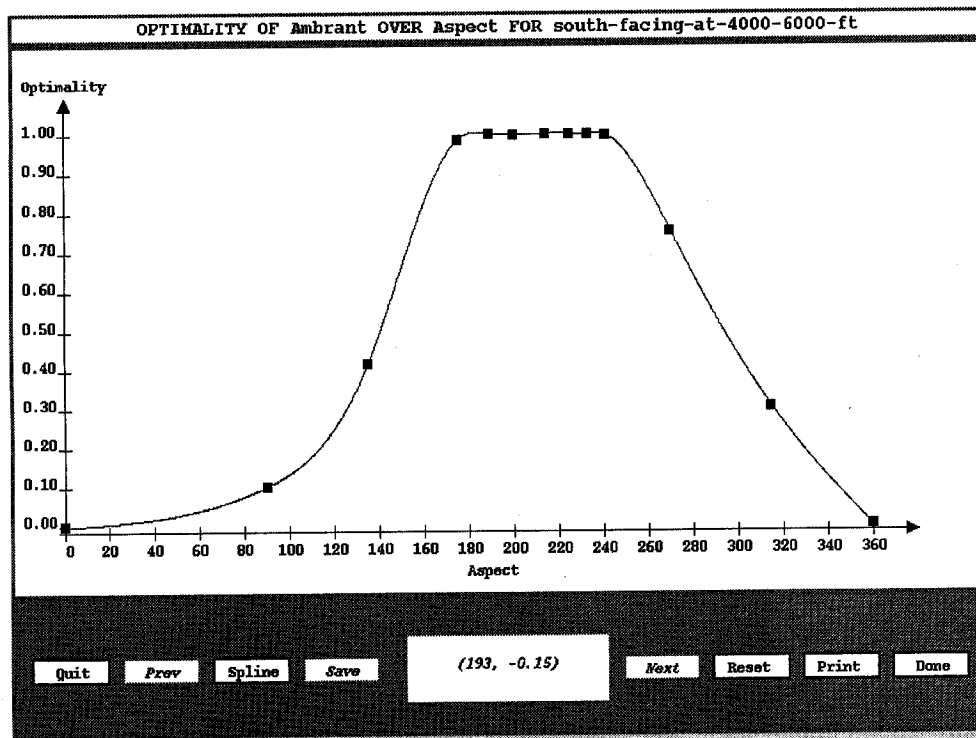


Fig. 2. Graphic user interface (GUI) for optimality definition.

Granite	Soil series
North facing	
> 4500 ft (1370 m)	
Gradient > 60%	Ovando
Gradient < 60%	Elkner
< 4500 ft (1370 m)	
Gradient > 60%	Rochester
Gradient < 60%	Ambrant
South facing	
> 6000 ft (1820 m)	
Gradient > 60%	Ovando
Gradient < 60%	Elkner
< 6000 ft (1820 m)	
Gradient > 60%	Rochester
Gradient < 60%	Ambrant

3.1.2. Soil–environment description interview

During the description interview, the soil experts were asked to describe the environmental configurations under which each soil series occurs. In other words, this description interview was designed to extract *Type 1* knowledge from the soil experts. During this interview, the knowledge engineer carefully checked the consistency of the soil experts as descriptions of very similar soil series can be confusing and soil experts might provide inconsistent environmental configuration. Thus, the knowledge engineer used the key developed during the key development interview to check the description provided by the experts. If a conflict existed, the knowledge engineer would immediately point it out to the soil experts. The result from this interview was the description of environmental conditions (environmental configurations) for each soil series as shown in Table 1.

3.1.3. Optimality curve definition interview

The optimality curve definition interview was designed to extract the knowledge about the behaviours of a given soil series with respect to changes of environmental conditions. In other words, it was structured for the extraction of *Type 2* knowledge. As discussed in Section 2, the *Type 2* knowledge acquisition was done one variable at a time. For example, the soil experts were asked to answer the following question: ‘How does the optimality for

soil series Ambrant change with respect to change of elevation?’. To elicit an answer, we provided the soil experts with a graphic user interface (GUI) (Zhu and Band, 1992). Fig. 2 shows the GUI used for the *Type 2* knowledge acquisition. The soil experts expressed their knowledge through the GUI by specifying the critical points for the optimality curve, which were then fit with a spline. The experts could change the critical points to fit the curves to their conceptual view of the optimality for a given soil series with respect to change of a particular environmental condition. If the soil development environment was described by five environmental variables, there would be five spline functions. If there were two or more instances for a soil series, there would be two or more sets of curves where each set is used to describe an instance. Fig. 3A and B show the elevation and aspect optimality curves for instance 1 of Ambrant soil series and Fig. 4A and B show the elevation and aspect optimality curves for instance 2 of Ambrant soil series. The optimality curves for

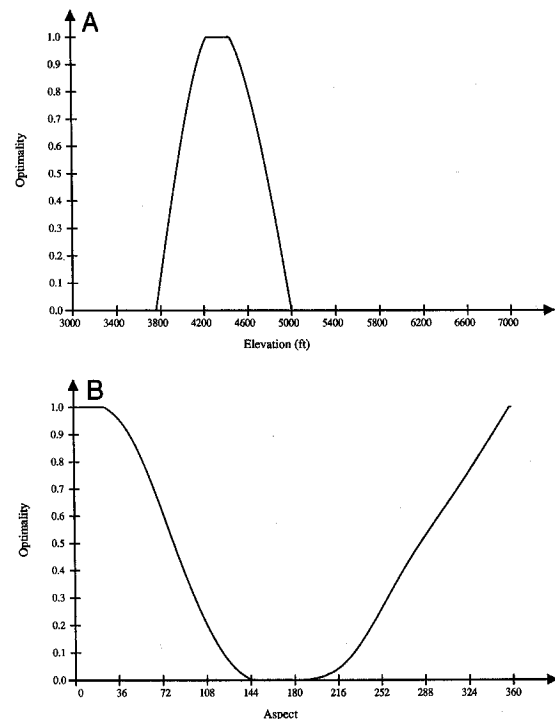


Fig. 3. Optimality curves for elevation (A) and aspect (B) of instance 1 of Ambrant soil series.

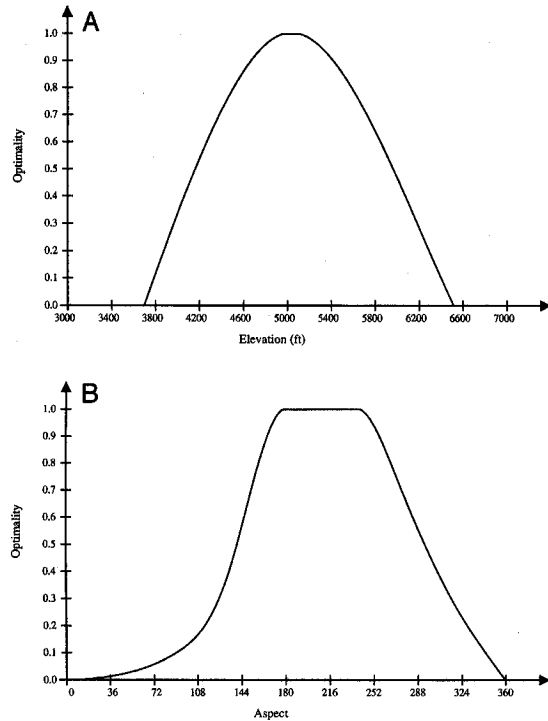


Fig. 4. Optimality curves for elevation (A) and aspect (B) of instance 2 of Ambrant soil series.

categorical variables such as parent materials are difficult to define using the above mentioned GUI. However, the influence of parent materials on soil series in the study area exhibits a crisp nature. The four soil series only occur on granite parent material and also no other soil series were found on granite material. Therefore, the optimality curve for parent materials is defined by a crisp function. For example, the optimality curve of parent materials for Ambrant is defined as

$$Op = \begin{cases} 1 & Pm \in \text{Granite} \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

where Op is the optimality and Pm is the parent material.

3.1.4. Knowledge verification interview

Once the two types of knowledge have been acquired in preliminary form, the knowledge verification interview was carried out to refine the knowledge set. The knowledge set was verified in two

parts: indoor and outdoor. During the indoor verification, the experts were provided with the results (images of s^k) from the system and compared these results against their perception (or mental map) of the given soil series. If the experts were unhappy with the results, they were allowed to modify the knowledge base and new results were produced. Soil experts again compared the new results with their mental perception. This process continued until the soil experts were satisfied with the results or the experts required outdoor verification (field checking) to clarify the difficult parts of the knowledge base. During the outdoor verification, the soil experts visited the area where the system had problems making correct inference. Information from field visits were also incorporated into the knowledge base. The process continued until a reasonable inference was made for these problematic areas.

3.2. Fuzzy soil inference

As discussed in Section 2, the fuzzy minimum operator was used to overlay the derived optimality curves and to obtain fuzzy membership values (similarity values) for all locations for given soil series (s^k). It was easy to apply the fuzzy minimum operator for those soil series which have only one instance because the value derived from the fuzzy minimum operator is the membership value for the given point. However, it was a little more complex to derive the membership value for soil series with more than one instance. The fuzzy maximum operator was used to derive the membership value in this situation. In other words, we chose the maximum value from these instances to represent the similarity to a given soil series. For example, Ambrant soil series exists either on north facing slopes with elevation from 4000 ft (1219.2 m) to 4500 ft (1371.6 m) (instance 1) or south facing slopes with elevation ranging from 4000 ft to 6000 ft (1828.8 m) (instance 2). For a point which is on north facing slope and at low elevation, it is certain that the membership value from instance 1 is greater than the value from instance 2. The membership value from instance 1 is more appropriate to represent the similarity of the soil at this point to Ambrant soil series than the membership value from instance 2.

The inference process was carried out using a raster data model such that fuzzy membership scores were computed for each grid cell. For a given soil series the inference system took a set of environmental conditions of a pixel from the GIS database. It then used each of the optimality curves to calculate the optimality value from each of the environmental variables. The fuzzy minimum operator then was used on these optimality values to obtain the membership value for the pixel. If the soil series had more than one instance, the environmental conditions were used again to calculate another set of optimality values according to the behaviours (the set of optimality curves) of the soil series defined for that instance. After all instances were exhausted, the fuzzy maximum operator was applied to the set of membership values obtained from the set of instances for that specific soil series and the resultant value from the maximum operator was considered the final membership value for the pixel. The process continued onto the next pixel until all pixels in the area were visited. A map (s^k) of membership values for the given soil series over the study area was then

produced. The process continued onto the next soil series until all soil series were exhausted and then the SSV values over the area were produced.

It should be noticed that although spatial dependency was not incorporated into the inference process, the spatial dependency was implicitly represented in the input environmental data and the optimality curves.

4. Study area and environmental variables employed

4.1. Study area

The study area for testing our methodology is the south east part of the Lubrecht Experimental Forest located about 50 km northeast of Missoula, Montana, USA. The study area is centred around North Fork of Elk Creek. The elevation in the study area ranges from 1130 m to 1950 m with high elevation in the south and low elevation in the north (Fig. 5). Ross and Hunter (1976) estimated that the mean annual

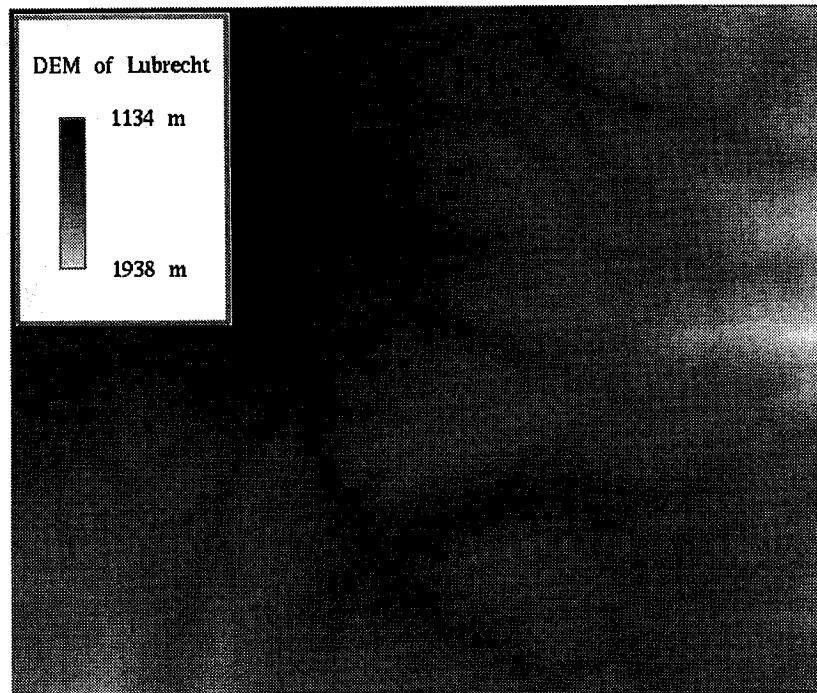


Fig. 5. Digital elevation model of the Lubrecht experimental forest.

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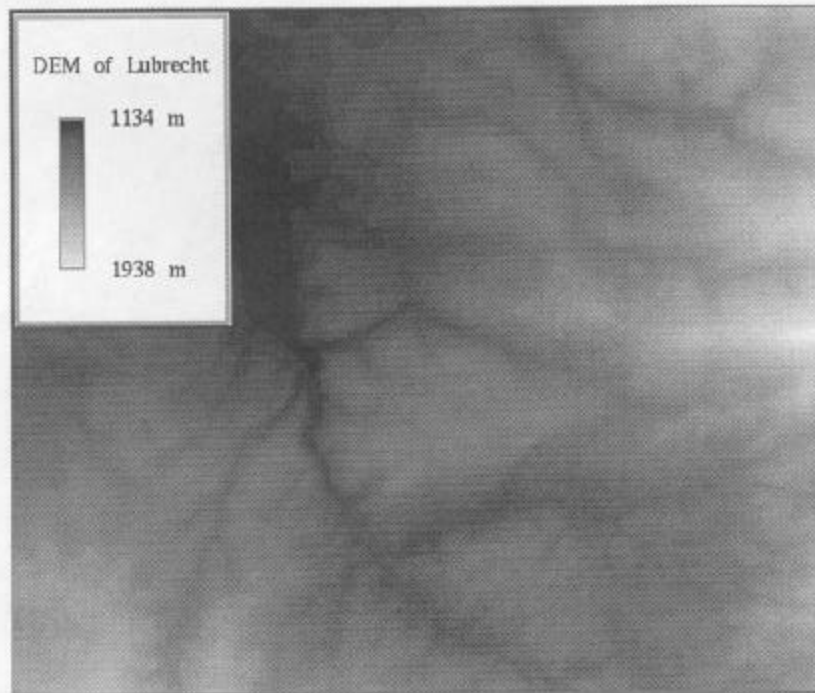


Fig. 5. Digital elevation model of the Lubrecht experimental forest.

precipitation for the Lubrecht area is between 50 and 76 cm. Approximately 44% of the precipitation falls during the winter (November through March) and 24% falls during the summer (June through August) (Nimlos, 1986). Most of the mountain slopes in the study area are forested, dominated by Douglas-fir (*Pseudotsuga menziesii*) although lesser amounts of western larch (*Larix occidentalis*) and ponderosa pine (*Pinus ponderosa*) are present. A small portion of the study area (the northwest) is covered by fescue/bluebunch wheatgrass. There are five types of parent materials in the area: Belt rocks, Granite, Limestone, Tertiary sediments, and Transported materials (Nimlos, 1986). The transported materials are lacustrine sediments and recent alluvium with the former in the northwest part and the latter distributed along the river streams in the study area. Landform-parent material associations are particularly strong in the study area. Landforms on Belt rocks, granite, and limestone are relatively steep mountainside slopes. Landforms on Tertiary-age sediments are gently undulating benches with low slope gradients. Alluvial and lacustrine deposits are nearly flat.

There are four soil orders in the study area: Alfisols, Entisols, Inceptisols, and Mollisols (Nimlos, 1986) and 22 mapped soil series. In this paper, we only discuss the inference of four soil series: Ambrant, Elkner, Ovando, and Rochester. These soil series are present only in the southern part of the Lubrecht Experimental Forest (Fig. 6). The characteristics of these soil series are shown in the Appendix (Missoula County Soil Survey, 1983).

4.2. Environmental variables used

This study employed elevation, parent material, aspect, canopy coverage, gradient, and surface profile curvature to characterize the soil forming environment. It may be noticed that in the data variable list there are no data variables which directly measure climatic factors. Although the study was conducted on a small drainage basin, great differences in terms of micro-climate do exist within the basin. However, these differences in micro-climate are well expressed by variations in elevation, aspect and gra-

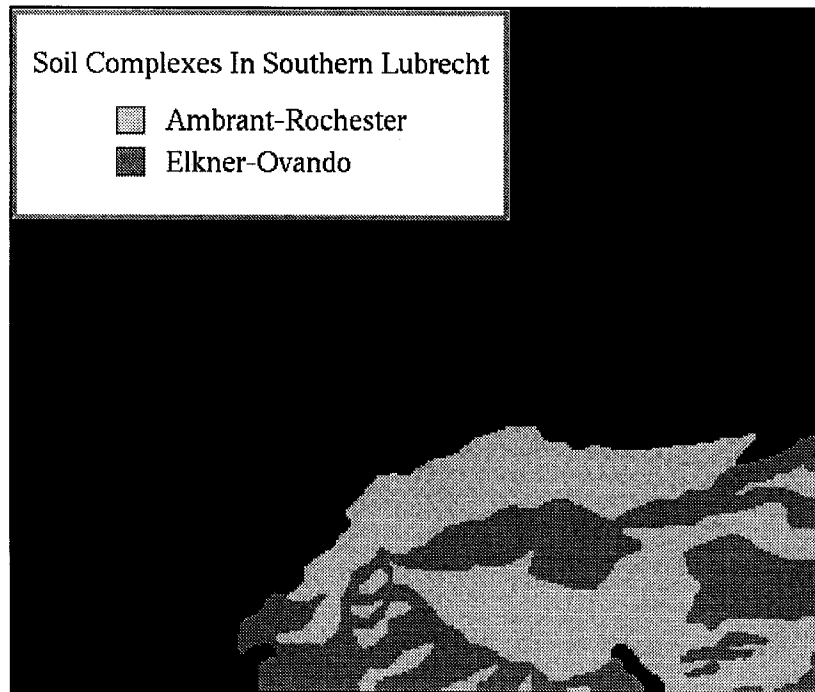


Fig. 6. Soil map of Ambrant–Rochester and Elkner–Ovando complexes.

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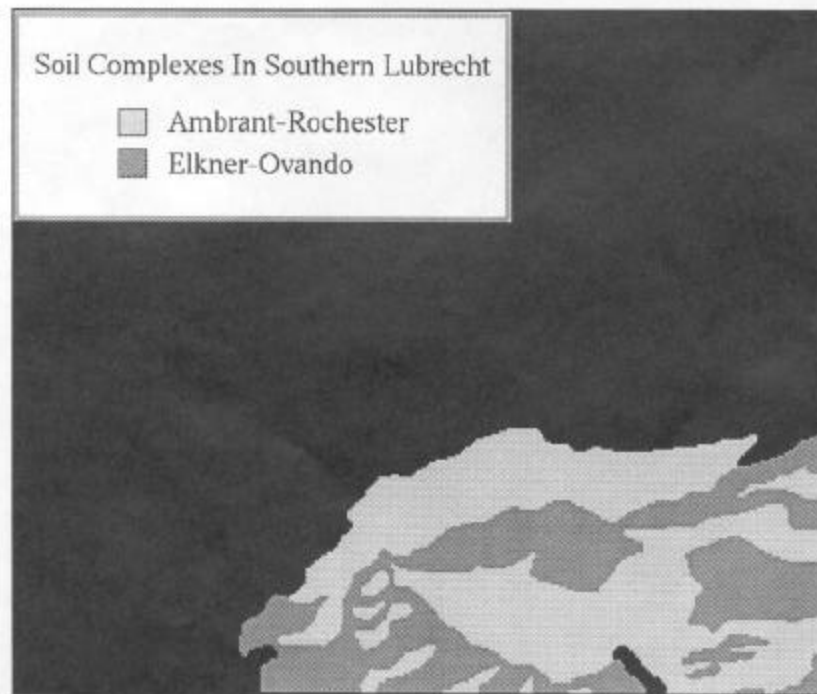


Fig. 6. Soil map of Ambrant-Rochester and Elkner-Ovando complexes.

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Information on elevation, aspect, gradient, and surface profile curvature were obtained from a digital elevation model (DEM) of the study area. The DEM was supplied by the GIS Laboratory of School of Forestry, University of Montana (Fig. 5). The accuracy of the DEM is comparable with the accuracy of the level 1 USGS 7.5-min DEMs (U.S. Geological Survey, 1990). The aspect (Fig. 7) and gradient (Fig. 8) were generated with a third-order finite difference method (Horn, 1981). The surface profile curvature (Fig. 9) was calculated based on the method of Zevenbergen and Thorne (1987).

Canopy coverage was approximated with an index derived from remotely sensed data (Thematic Mapper). Recent research suggests that reasonable estimates of canopy closure can be gained with middle infrared wavelengths (*MIR*) (Butera, 1986; Baret et al., 1988). Nemani et al. (1993) have used the changes in *MIR* (TM band 5, 1.55–1.75 μm) response to canopy closure in combination with red to infrared

ratios to estimate leaf area index (LAI) in the study site. In this study, we use the *MIR* to estimate an index of canopy cover

$$CC = 100 \left(1 - \frac{MIR - MIR_{\min}}{MIR_{\max} - MIR_{\min}} \right) \quad (7)$$

where *CC* stands for canopy coverage index, MIR_{\min} and MIR_{\max} are middle infrared radiances from completely closed and completely open canopies in/around the study area, respectively. Note that Eq. 7 was not correlated with actual ground estimates of canopy cover and therefore should be considered an index, rather than an estimate of actual percentage canopy cover. The reflectance data were acquired from the LANDSAT Thematic Mapper on 16 July 1984. The TM (Thematic Mapper) Band 5 was preprocessed for topographic correction (Eqs. 8 and 9) before it was used in Eq. 7. Eq. 8 is based on the equation given by Civco (1989):

$$CDN = DN + DN \left[(\bar{I} - I) / \bar{I} \right] \quad (8)$$

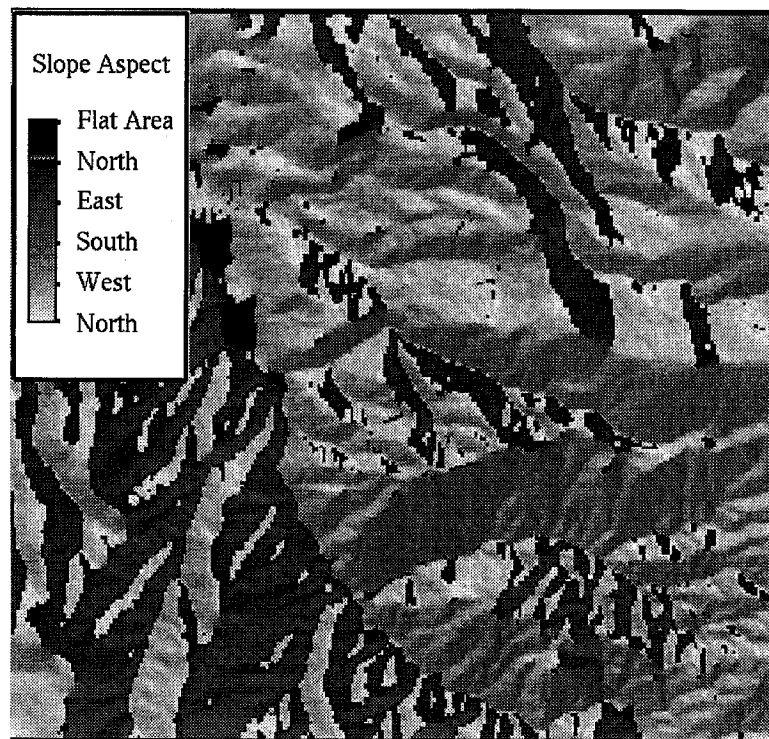


Fig. 7. Slope aspect of study area.

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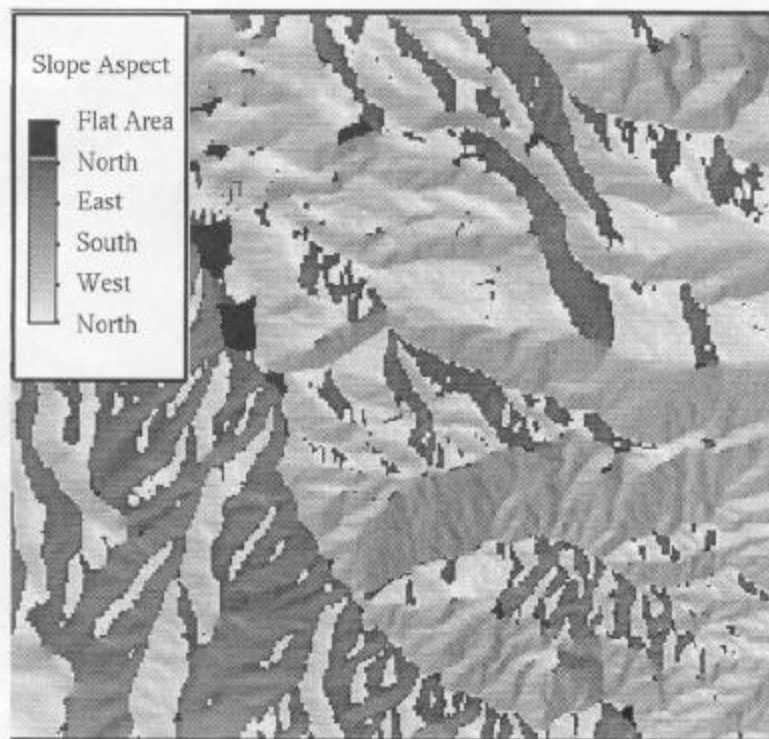


Fig. 7. Slope aspect of study area.

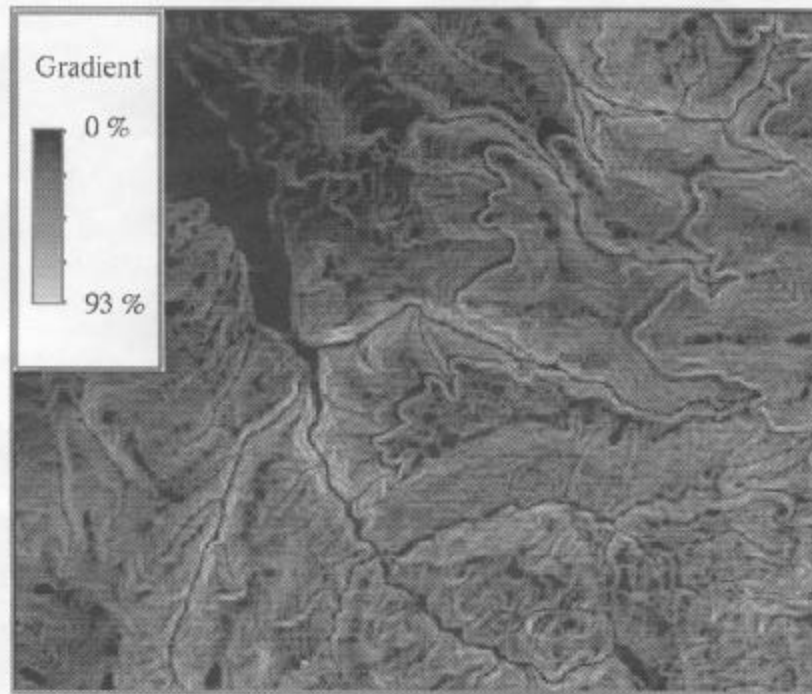


Fig. 8. Slope gradient of study area.

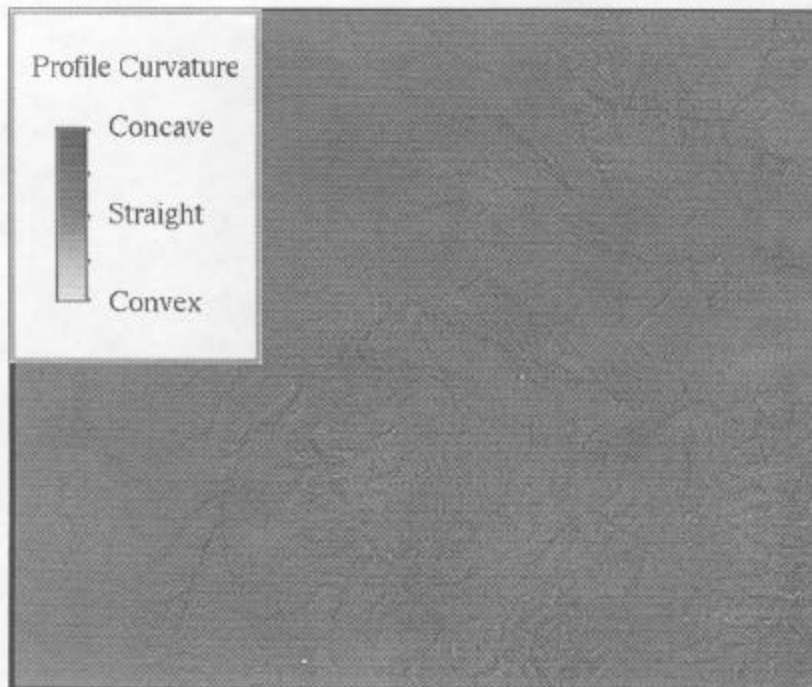


Fig. 9. Surface profile curvature of study area.

where CDN is the corrected digital number, DN is the original digital number, \bar{I} is the mean of the scaled (0 to 100) incidence values of the whole scene, and I is the scaled (0 to 100) incidence value at the location of that DN . I is calculated according to the following formula (Colby, 1991).

$$I = 100(\cos(\alpha) \cos(Z) + \sin(\alpha) \sin(Z) \cos(D)) \quad (9)$$

where α is the slope gradient (in degrees), Z is the zenith angle of the sun at the time of the image was taken, and D is the difference between the azimuth of sun and the slope aspect. Fig. 10 shows the canopy coverage of the study area.

Information on parent material was obtained from the geological map of the study area (Brenner, 1968) (Fig. 11). It was recognized that the inclusion of soil parent material from geological maps would also affect the performance of the system because geological maps potentially contain human errors.

These variables have by no means exhausted the soil formation factors and the interaction of these factors on soil development. They were used to

demonstrate the potential of the new methodology of soil information gathering and representation.

5. Results and discussion

The results from our method of soil inference can be presented and compared to the soil map in three aspects: spatial patterns (spatial level), attribute details (attribute level), and specific points (point level). The existing soil map contains only the complex of Ambrant–Rochester and the complex of Elkner–Ovando. In order to compare the results from the new methodology presented in this paper with the existing soil map, we derived the fuzzy membership map for the Ambrant–Rochester complex from the membership map of Ambrant soil series and that of Rochester soil series using fuzzy maximum operator. The same procedure was applied to the membership map of Elkner and that of Ovando for the creation of the fuzzy membership map of Elkner–Ovando complex.

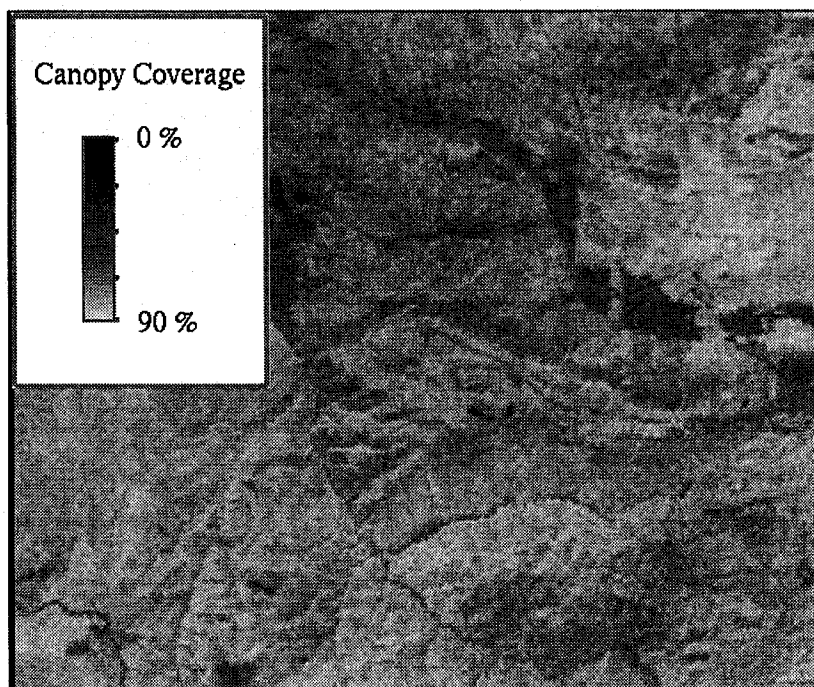


Fig. 10. Canopy coverage index of study area.

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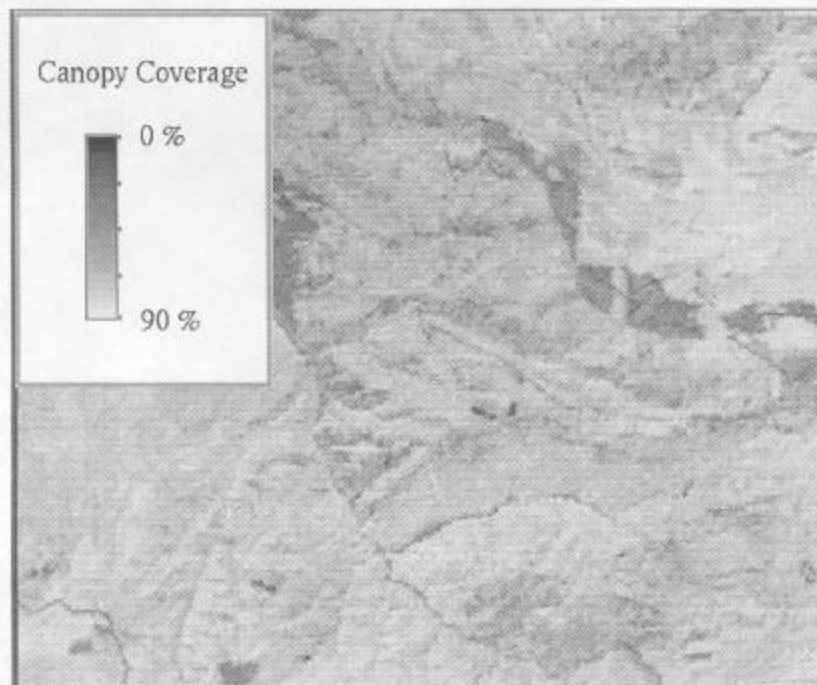


Fig. 10. Canopy coverage index of study area.

Fig. 12A and B show the distribution of fuzzy membership values and soil map, respectively, for the Ambrant–Rochester complex. Fig. 13A and B show the distribution of fuzzy membership values for the Elkner–Ovando complex and the distribution of this complex on a soil map, respectively. Visual inspection indicates that Fig. 12A and B, and Fig. 13A and B show very similar spatial patterns, respectively. There are, however, apparent differences between the membership maps and the soil maps. The main difference is that the membership maps reveal more details at the spatial level. Using a raster-based, fuzzy logic mapping, the degree of similarity of soil series allows much greater spatial resolution than is feasible using a polygon-based, binary system (in or out of a set). The general shapes for the soil complexes on the membership images follow the landscape better than the ones on the soil maps where inclusion or exclusion from a region is based more on restrictions deriving from the scale of the map than on local conditions. On the soil maps, areas

mapped as belonging to these soil complexes are necessarily larger and more generalized mapping features. However, on the membership images, not all pixels within an area have high possibility values, which means that the similarity of the local soil to the central concept of the series responds to local variations in the apparent soil forming environment, as can be observed in the field. Therefore, our method is capable of eliminating the minimum mapping size problem in conventional soil mapping and by allowing more detail spatial patterns of soil information to be represented.

At the attribute level, on the soil maps, the soil at a location is assigned to one and only one soil series. In this case, the expected properties for the soil at that location can only be set as the expected properties for the assigned series with no transitional properties between soils dependent on landscape and environmental conditions. From our method, the soil at a location is presented as a SSV (soil similarity vector) with each of its elements representing infor-

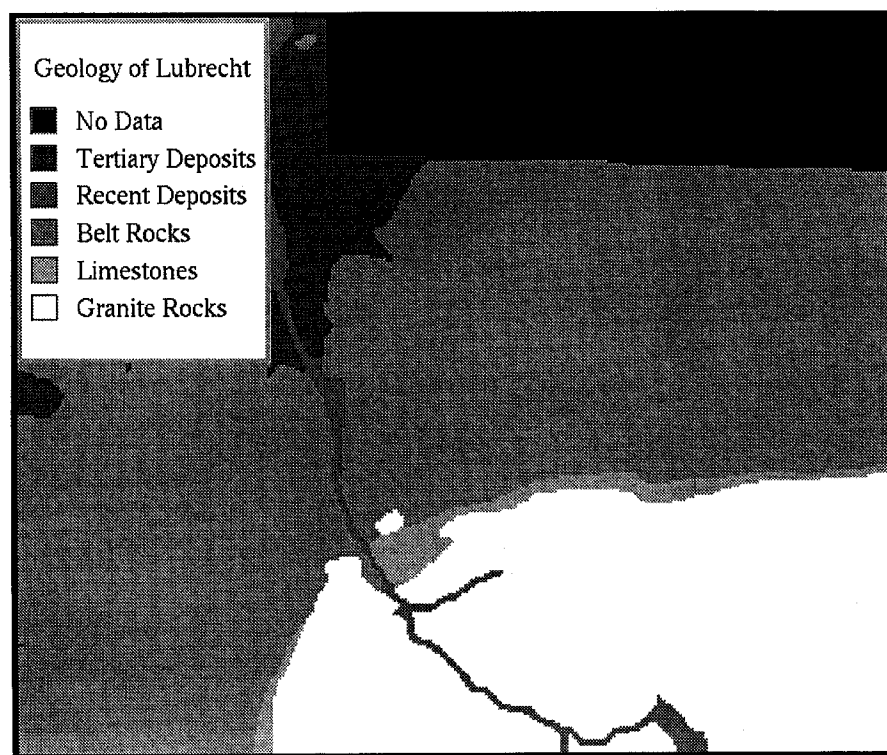


Fig. 11. Lithology of study area.

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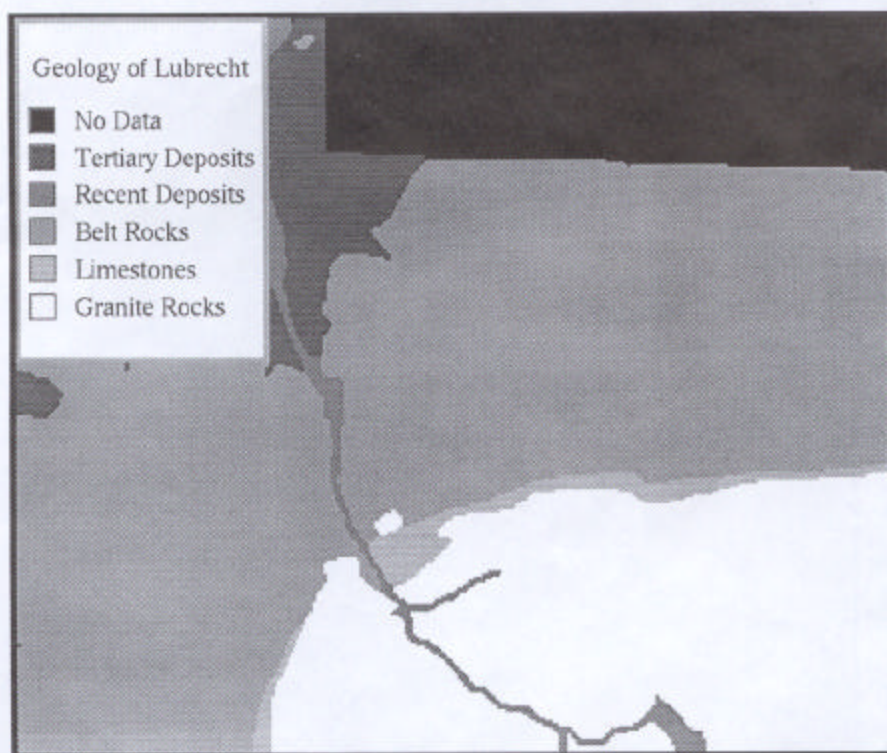


Fig. 11. Lithology of study area.

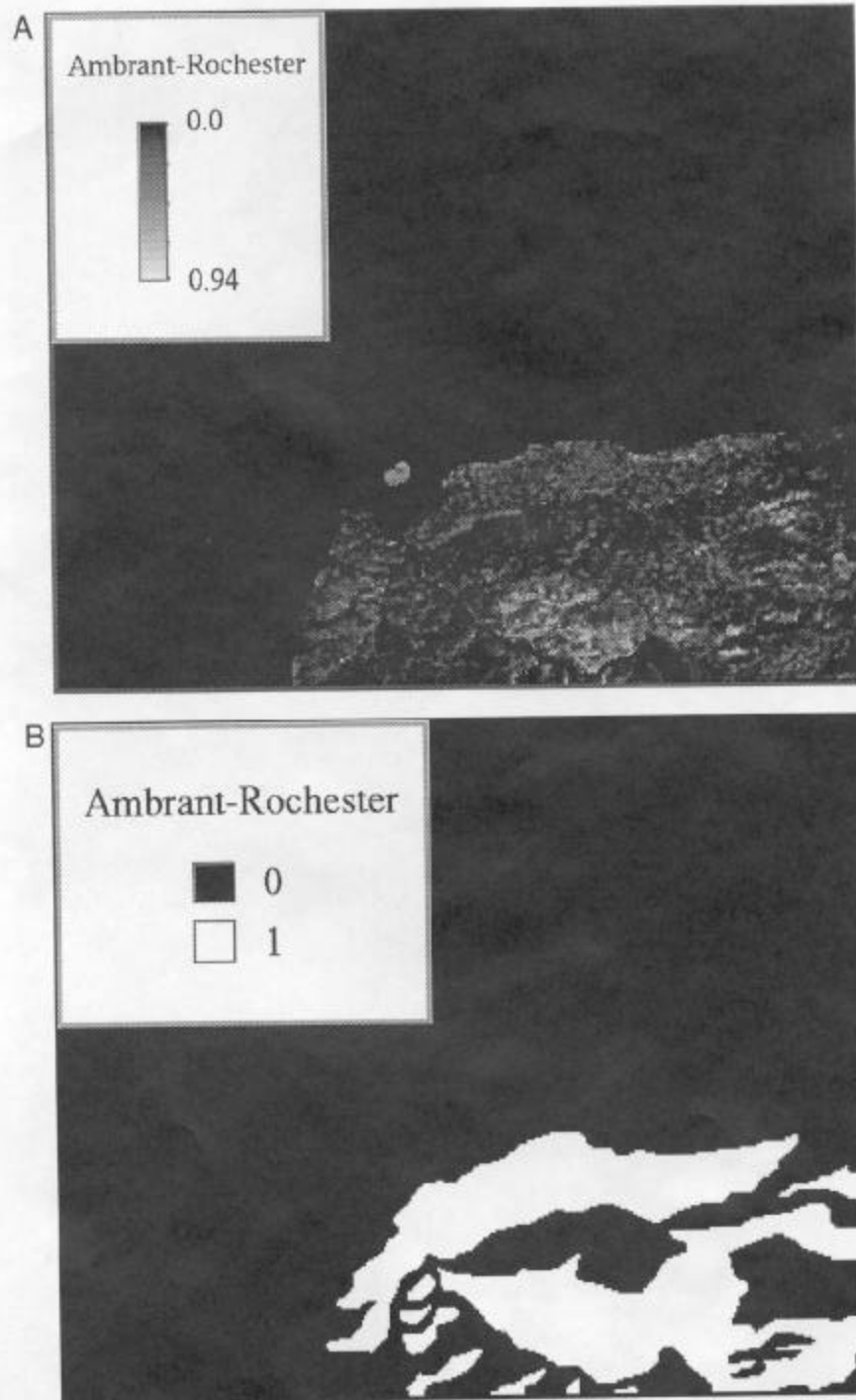


Fig. 12. (A) Membership map of Ambrant-Rochester complex. (B) Soil survey map of Ambrant-Rochester complex.

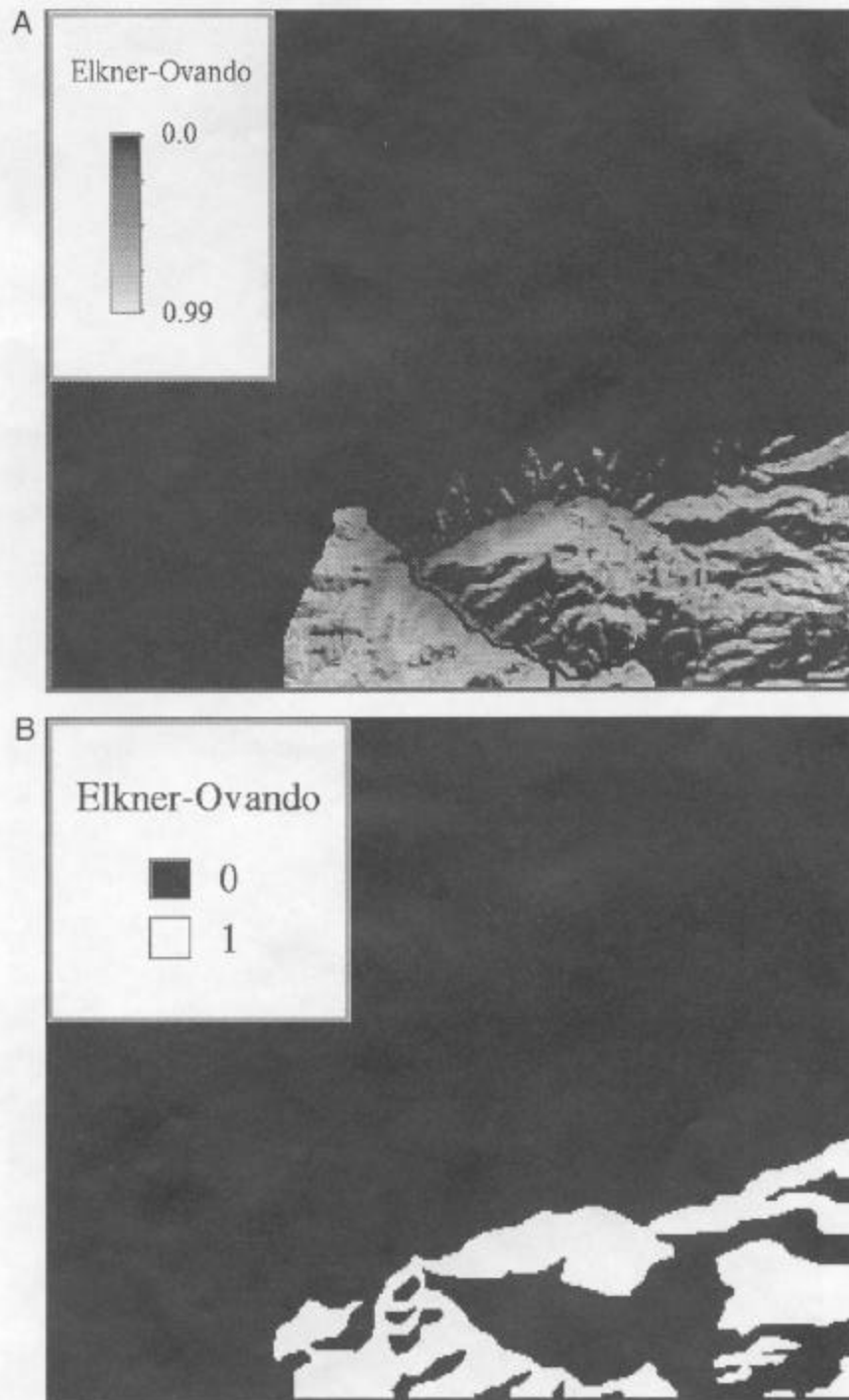


Fig. 13. (A) Membership map of Elkner–Ovando complex. (B) Soil survey map of Elkner–Ovando complex.

Table 2
SSV values for points on the NW–SE transect shown in Fig. 14

Point	Ambrant	Rochester	Elkner	Ovando
1	0.2644	0.0210	0.3541	0.3605
2	0.4595	0.0000	0.1746	0.3659
3	0.3824	0.0669	0.2467	0.3040
4	0.1617	0.0281	0.3630	0.4472
5	0.1250	0.0000	0.4928	0.3822
6	0.5239	0.3947	0.0310	0.0504
7	0.0296	0.0000	0.5389	0.4314
8	0.7156	0.0000	0.2844	0.0000
9	0.3953	0.0363	0.4628	0.1054
10	0.1898	0.1017	0.4647	0.2437
11	0.4958	0.5025	0.0011	0.0006
12	0.4901	0.4257	0.0402	0.0439
13	0.0403	0.0000	0.4406	0.5192
14	0.0708	0.0000	0.6273	0.3020

Points in the table are about 180 m apart on the transect (NW–SE) starting from NW.

mation of the degree of similarity to each of the candidate soil series (Tables 2 and 3, Fig. 14). It may then be possible to infer soil properties intermediate to the finite set of candidate soil series and thus

Table 3
SSV values for points on the SW–NE transect shown in Fig. 14

Point	Ambrant	Rochester	Elkner	Ovando
1	0.1330	0.0000	0.6264	0.2407
2	0.0362	0.0000	0.5202	0.4436
3	0.2053	0.1226	0.3334	0.3387
4	0.4665	0.0000	0.4451	0.0884
5	0.5506	0.4457	0.0025	0.0012
6	0.5378	0.4474	0.0045	0.0103
7	0.7857	0.0983	0.1160	0.0000
8	0.6718	0.1527	0.1165	0.0591
9	0.1554	0.0466	0.4763	0.3216
10	0.0878	0.0000	0.5861	0.3261
11	0.1115	0.0000	0.6845	0.2040
12	0.0452	0.0000	0.4862	0.4686
13	0.9791	0.0000	0.0000	0.0209
14	0.6694	0.0000	0.1116	0.2190
15	0.2852	0.0000	0.6248	0.0900
16	0.1280	0.0122	0.4233	0.4365
17	0.1261	0.0000	0.4632	0.4106
18	0.1931	0.0000	0.7076	0.0993
19	0.0846	0.0000	0.5196	0.3958
20	0.1518	0.0000	0.3918	0.4563

Points in the table are about 180 m apart on the transect (SW–NE) starting from SW.

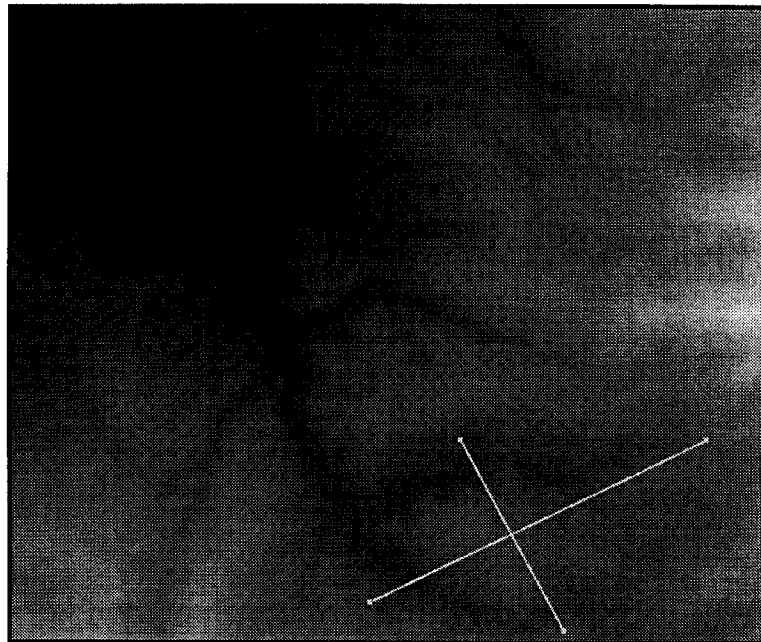


Fig. 14. Location of the two transects.

Table 2
SSV values for points on the NW–SE transect shown in Fig. 14

Point	Ambrant	Rochester	Elkner	Ovando
1	0.2644	0.0210	0.3541	0.3605
2	0.4595	0.0000	0.1746	0.3659
3	0.3824	0.0669	0.2467	0.3040
4	0.1617	0.0281	0.3630	0.4472
5	0.1250	0.0000	0.4928	0.3822
6	0.5239	0.3947	0.0310	0.0504
7	0.0296	0.0000	0.5389	0.4314
8	0.7156	0.0000	0.2844	0.0000
9	0.3953	0.0363	0.4628	0.1054
10	0.1898	0.1017	0.4647	0.2437
11	0.4958	0.5025	0.0011	0.0006
12	0.4901	0.4257	0.0402	0.0439
13	0.0403	0.0000	0.4406	0.5192
14	0.0708	0.0000	0.6273	0.3020

Points in the table are about 180 m apart on the transect (NW–SE) starting from NW.

mation of the degree of similarity to each of the candidate soil series (Tables 2 and 3, Fig. 14). It may then be possible to infer soil properties intermediate to the finite set of candidate soil series and thus

Table 3
SSV values for points on the SW–NE transect shown in Fig. 14

Point	Ambrant	Rochester	Elkner	Ovando
1	0.1330	0.0000	0.6264	0.2407
2	0.0362	0.0000	0.5202	0.4436
3	0.2053	0.1226	0.3334	0.3387
4	0.4665	0.0000	0.4451	0.0884
5	0.5506	0.4457	0.0025	0.0012
6	0.5378	0.4474	0.0045	0.0103
7	0.7857	0.0983	0.1160	0.0000
8	0.6718	0.1527	0.1165	0.0591
9	0.1554	0.0466	0.4763	0.3216
10	0.0878	0.0000	0.5861	0.3261
11	0.1115	0.0000	0.6845	0.2040
12	0.0452	0.0000	0.4862	0.4686
13	0.9791	0.0000	0.0000	0.0209
14	0.6694	0.0000	0.1116	0.2190
15	0.2852	0.0000	0.6248	0.0900
16	0.1280	0.0122	0.4233	0.4365
17	0.1261	0.0000	0.4632	0.4106
18	0.1931	0.0000	0.7076	0.0993
19	0.0846	0.0000	0.5196	0.3958
20	0.1518	0.0000	0.3918	0.4563

Points in the table are about 180 m apart on the transect (SW–NE) starting from SW.

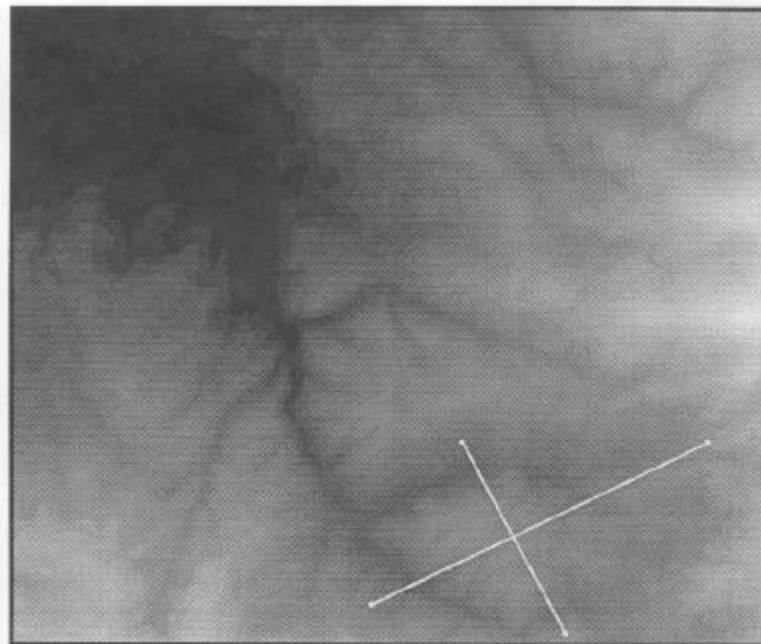


Fig. 14. Location of the two transects.

Table 4
Hardened results and mapped soils for points in Tables 2 and 3

Hardened from Table 2			Hardened from Table 3		
Point	Hardened as	On soil map	Point	Hardened as	On soil map
1	Elkner–Ovando	Ambrant–Rochester	1	Elkner–Ovando	Elkner–Ovando
2	Ambrant–Rochester	Ambrant–Rochester	2	Elkner–Ovando	Elkner–Ovando
3	Ambrant–Rochester	Elkner–Ovando	3	Elkner–Ovando	Elkner–Ovando
4	Elkner–Ovando	Elkner–Ovando	4	Ambrant–Rochester	Elkner–Ovando
5	Elkner–Ovando	Elkner–Ovando	5	Ambrant–Rochester	Ambrant–Rochester
6	Ambrant–Rochester	Elkner–Ovando	6	Ambrant–Rochester	Ambrant–Rochester
7	Elkner–Ovando	Elkner–Ovando	7	Ambrant–Rochester	Ambrant–Rochester
8	Ambrant–Rochester	Ambrant–Rochester	8	Ambrant–Rochester	Ambrant–Rochester
9	Elkner–Ovando	Ambrant–Rochester	9	Elkner–Ovando	Elkner–Ovando
10	Elkner–Ovando	Ambrant–Rochester	10	Elkner–Ovando	Elkner–Ovando
11	Ambrant–Rochester	Ambrant–Rochester	11	Elkner–Ovando	Elkner–Ovando
12	Ambrant–Rochester	Ambrant–Rochester	12	Elkner–Ovando	Elkner–Ovando
13	Elkner–Ovando	Elkner–Ovando	13	Ambrant–Rochester	Ambrant–Rochester
14	Elkner–Ovando	Elkner–Ovando	14	Ambrant–Rochester	Ambrant–Rochester
			15	Elkner–Ovando	Elkner–Ovando
			16	Elkner–Ovando	Elkner–Ovando
			17	Elkner–Ovando	Elkner–Ovando
			18	Elkner–Ovando	Elkner–Ovando
			19	Elkner–Ovando	Elkner–Ovando
			20	Elkner–Ovando	Elkner–Ovando

approximate the continuum of soil properties over the landscape. At this point, this latter inference of intermediate soil properties is under investigation.

In order to compare the inferred results with soil maps at the point level, we hardened the SSV to produce a crisp representation of soil information for the points in Tables 2 and 3. We assigned to the point the soil complex which has the highest membership value in the SSV for the point. For example, Point 1 and 2 in Table 2 are assigned as Elkner–Ovando and Ambrant–Rochester soil complexes, respectively. Table 4 lists the hardened results (inferred complexes) and the mapped soil complexes for the points in Tables 2 and 3. From Table 4, it can be seen that the inferred complexes in general agree with the mapped soil complexes for these points (for 28 of 34 points, the inferred complexes match the mapped complexes). Among the six points at which the inferred results do not match with the mapped information, four (except points 6 and 10 in Table 2) have membership distributed evenly over the two complexes. This could mean that the soil at these points do not strongly belong to any of these two complexes but demonstrate even similarities to these

two complexes. At points 6 and 10 in Table 2 the inferred results strongly disagree with the mapped information. The disagreement at these areas is currently under investigation.

6. Conclusions

This paper presents a methodology for soil information gathering and representation. The methodology consists of structured knowledge acquisition techniques, fuzzy inference techniques, and GIS techniques. The knowledge acquisition techniques were used to acquire knowledge about soil–environmental relationships. The GIS techniques were used to derive data on the environmental variables. A set of fuzzy inference techniques were employed to infer soil series based on the acquired knowledge and environmental data provided by the GIS techniques. The fuzzy membership images produced from the new method have potential advantages over standard soil survey maps in terms of revealing spatial patterns of soil information, detailing attribute information and in terms of production cost. Rigorous field

testing is required to quantify the potential advantages of this technique in the derivation and representation of the spatial pattern of soil types and properties, and is in progress at this point.

There are several research directions for improving this new methodology. At this stage, we were using a lithologic map as the parent material map. More research needs to be done to provide more accurate parent material maps. Secondly, in this illustration, only the primitive topographic indices (such as slope gradient, slope aspect, curvature, and elevation) were used. No geomorphic features (such as slopes, ridges, terraces, and bottom lands, etc.) were used at this point. It is obvious that the inclusion of these geomorphic features will further improve the quality of the inferred soil information. Additional and improved information on the vegetation canopy from remotely sensed data would allow the extension of the technique over larger areas. In addition, an expansion of the knowledge set to incorporate spatial dependency of soil information (such as catenary relationships) would provide the ability both to facilitate the inference of specific soil properties from the SSV, and to improve soil pattern prediction by providing further information on landscape/soil context.

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Appendix A. Characteristics of the four soil series

A.1. *Ambrant series*

Deep, somewhat excessively drained soils formed in colluvium derived from granite; the profile is neutral to medium acid.

The A horizon is sandy loam and contains 15 to 35% pebbles; light brownish grey (10YR 6/2); moderate fine granular structure; soft, very friable, nonsticky, and nonplastic.

The AB horizon is coarse sandy loam and contains 15 to 35% pebbles; light brownish grey (2.5Y 6/2); weak fine and medium blocky structure parting to moderate fine and medium granular structure; soft, very friable, nonsticky, and nonplastic.

The C horizon is coarse sand or loamy sand and contains 35 to 60% pebbles; light brownish grey (2.5Y 6/2); massive; slightly hard, very friable, nonsticky, and nonplastic.

A.2. *Elkner series*

Deep, somewhat excessively drained soils formed in colluvium derived from granite; the profile is medium acid or slightly acid.

The A horizon is sandy loam and contains 0 to 15% pebbles and 0 to 5% cobbles; pale brown (10YR 6/3); weak coarse granular structure; soft, very friable, nonsticky, and nonplastic.

The AB horizon is coarse sandy loam and contains 0 to 15% pebbles and 0 to 5% cobbles; light yellowish brown (10Y 6/4); weak coarse subangular blocky structure; slightly hard, very friable, nonsticky, and nonplastic.

The C horizon is loamy coarse sand and contains 15 to 25% pebbles and 0 to 10% cobbles; light yellowish brown (10YR 6/4); massive; loose, nonsticky, and nonplastic.

A.3. *Ovando series*

Deep, excessively drained soils formed in colluvium derived from granite; the profile is medium acid or slightly acid.

The A horizon is sandy loam and contains 15 to 35% pebbles and 0 to 1% stones; light yellowish brown (10YR 6/4); moderate very fine and fine granular structure; soft, very friable, nonsticky, and nonplastic.

The AB horizon is loamy coarse sand and contains 35 to 50% pebbles and 0 to 10% cobbles; very pale brown (10YR 7/4); single grain; loose, nonsticky, and nonplastic.

The C horizon is loamy coarse sand and contains 50 to 60% pebbles and 10 to 20% cobbles; pale brown (10YR 6/3); single grain; loose, nonsticky, and nonplastic.

A.4. Rochester series

Deep, excessively drained soils formed in colluvium derived from granite; the profile is neutral.

The A horizon is sandy loam and contains 15 to 35% pebbles; light brownish grey (2.5Y 6/2); weak coarse granular structure; soft, very friable, non-sticky, and nonplastic.

The C horizon is loamy coarse sand or coarse sand and contains 35 to 60% pebbles; light brownish grey (2.5Y 6/2); weak medium and coarse subangular blocky structure; soft, very friable, nonsticky, and nonplastic.

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